Efficient algorithms for average completion time scheduling

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Abstract

We analyze the competitive ratio of algorithms for minimizing (weighted) average completion time on identical parallel machines and prove that the well-known shortest remaining processing time algorithm (SRPT) is 5/4-competitive w.r.t. the average completion time objective. For weighted completion times we give a deterministic algorithm with competitive ratio $1.791 + o(m)$. This ratio holds for preemptive and non-preemptive scheduling.

1 Introduction

There is a vast amount of papers on minimizing average completion in machine scheduling. Most appeared in the combinatorial optimization community in the last fifteen years. The papers by Schulz and Skutella [20] and Correa and Wagner [6] give a good overview.

The shortest remaining processing time (SRPT) algorithm is a well-known and simple online procedure for preemptive scheduling of jobs. It produces an optimal schedule on a single machine with respect to the average completion time objective [18]. The example in Figure 1 shows that this is not true when SRPT is applied to parallel machines. The best known upper bound on its competitive ratio was 2 [16] until recently (SODA2010), Chung et al. [5] showed that the ratio is at most 1.86. Moreover, they show that the ratio is not better than $21/19 > 1.105$. In this paper, we show that the competitive ratio of SRPT is at most 1.25.

The SRPT algorithm has a natural generalization to the case where jobs have given weights. Unfortunately, our proof does not carry over to this case. No algorithm is known to have a competitive ratio less than 2. Remarkably, even for the offline problem, the only ratio less than 2 results from the approximation scheme given by Afrati et al. [1]. Schulz and Skutella [20] give a randomized 2-approximate algorithm which can be derandomized and applied online (although not at the same time). A deterministic online algorithm for the preemptive case is given by Megow and Schulz [14] and for the non-preemptive case by Correa and Wagner [6]. The ratios are, respectively, 2 and 2.62. The first bound on the algorithm is tight, the latter is probably not. On the single machine, no non-preemptive online algorithm can be better than 2 competitive [25] but it was unknown if the same is

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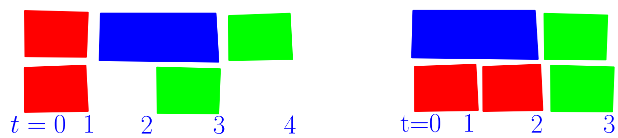

Figure 1: There are two machines. At time 0, two jobs of length 1 and one job of length 2 are released and at time 2, two jobs of length 1 are released. The picture shows the suboptimal SRPT schedule and the optimal schedule.

true for parallel machines. We give a simple online algorithm that runs in $O(n \log n)$ time and has competitive ratio $1.791 + o(m)$, i.e., it drops down to 1.791 for $m \to \infty$. This gives new insight in online and offline algorithms for average completion time minimization on parallel machines.

The first approximation guarantee for weighted non-preemptive scheduling was given by Hall et al. [9]. This ratio of $4 + \epsilon$ was reduced to 3.28 by Megow and Schulz [14] and then reduced to 2.62 by Correa and Wagner [6]. Table 1 gives a summary of known best ratios for a selection of problems. Remarkable is the large gap between lower and upper bounds for parallel machines. Not mentioned in the table are recent papers by Jaillet and Wagner [11] and by Chou et al. [4] which analyze the asymptotic ratio for several of these problems. Asymptotic, in this case, means that jobs have comparable weights and the number of jobs goes to infinity.

1.1 Problem definition

An instance is given by a number of machines m, a job set $J \subset \mathbb{N}$ and for each $j \in J$ integer parameters $p_j \geq 1, r_j \geq 0, w_j \geq 0$ indicating the required processing time, the release time, and the weight of the job. A schedule is an assignment of jobs to machines over time such that no job is processed by more than one machine at the time and no machine processes more than one job at the time. In the non-preemptive setting, each job j is assigned to one machine and is processed without interruption. In the preemptive setting, we may repeatedly interrupt the processing of a job and continue it at any time on any machine. The algorithm has to construct the schedule online, i.e., the number of machines is known a priori but jobs are only revealed at their release times . Even the

Problem (Online)	L.B. Rand.		U.B. Rand.		L.B. Det.		U.B. Det.	
$1\|r_j, pmtn\|\sum_j C_j$	1		1	[18]	1		1	[18]
$1\|r_j, pmtn\|\sum_j w_j C_j$	1.038	[23]	4/3	[19]	1.073	[23]	1.57	[22]
$1\|r_j\|\sum_j C_j$	$e/(e-1)$	[24]	$e/(e-1) \approx 1.58$	[3]	2	[25]	2	[10][13][16]
$1\|r_j\|\sum_j w_j C_j$	$e/(e-1)$	[24]	1.69	[8]	2	[25]	2	[2][17]
$P\|r_j, pmtn\|\sum_j C_j$	1	–	$1.86 \to 5/4$	[5]	1.047	[25]	$1.86 \to 5/4$	[5]
$P\|r_j, pmtn\|\sum_j w_j C_j$	1	–	$2 \to 1.791$	[20][14]	1.047	[25]	$2 \to 1.791$	[14]
$P\|r_j\|\sum_j C_j$	1.157	[21]	$2 \to 1.791$	[20]	1.309	[25]	$2 \to 1.791$	[12]
$P\|r_j\|\sum_j w_j C_j$	1.157	[21]	$2 \to 1.791$	[20]	1.309	[25]	$2.62 \to 1.791$	[6]

Table 1: Known lower and upper bounds on the competitive ratio for randomized and deterministic online algorithm.

The SRPT algorithm:

Let $t = 1$. Repeat:
If there are more than m jobs available for slot t, then process m jobs in slot t that have the shortest remaining processing times among all available jobs. Otherwise, process all available jobs. Let $t = t + 1$.

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number of jobs $n = |J|$ is unknown until the last job has been scheduled. Given a schedule, we denote the completion time of job j by C_j. The value of a schedule is the weighted average completion time $\frac{1}{n}\sum_{j\in J} w_j C_j$ and the objective is to find a schedule with small value. We say that an algorithm is c-competitive if it finds for any instance a schedule with value at most c times the optimal value.

2 The competitive ratio of SRPT

Phillips et al. [16] showed that *SRPT* is at most 2-competitive and showed that their analysis is tight. Hence, a new idea is needed to prove a smaller ratio. Indeed, the proof by Chung et al [5] is completely different and uses a sophisticated randomized analysis of the optimal solution. On the contrary, our proof builds on the original proof of Phillips et al. and continues where that proof stops. Their main lemma is one of the four lemmas in our proof (Lemma 2).

In the proof, we may restrict ourselves to schedules that preempt jobs only at integer time points since all processing times and release times are integer. For any integer $t \geq 1$ we define *slot* t as the time interval $[t-1, t]$. By this notation, the first slot that is available for j is slot $r_j + 1$. Given a (partial) schedule σ, we say that job j is *unfinished* at time t (or, equivalently, *unfinished* before slot $t + 1$) if less than p_j units are processed before t in σ. A job j is *available* at time t (or, equivalently, *available* for slot $t+1$) if $r_j \leq t$ and j is unfinished at time t. Let $\sigma(t)$ be the set of jobs processed in slot t and denote by $\mu_i(\sigma)$ the i-th smallest completion time in σ.

The SRPT algorithm as defined here is not deterministic since it may need to choose between jobs with the same remaining processing time. We say that a schedule σ is an *SRPT schedule* for instance I if it is a possible output of the SRPT algorithm applied to I. Note that the values $\mu_i(\sigma)$ do not depend on the non-deterministic choices of the algorithm, i.e., if σ and σ' are SRPT schedules for the same instance on n jobs, then $\mu_i(\sigma) = \mu_i(\sigma')$ for all $i \in \{1, 2, \ldots, n\}$.

All four lemmas are quite intuitive. For the first lemma, imagine that for a given instance we reduce the release time of some job by δ and increase its processing time by at least the same amount. Then, the optimum value cannot improve since there is no advantage in starting a job earlier if this is undone by an increase in its processing time. The first lemma shows that SRPT has an even stronger property in this case. The proof is given in the appendix.

Lemma 1 *Let I and I' satisfy $J = J'$ and for each $j \in J$ satisfy $r'_j = r_j - \delta_j \geq 0$ and $p'_j \geq p_j + \delta_j$, for some integers $\delta_j \geq 0$. Let σ and σ' be SRPT schedules for, respectively,*

I and I'. Then, for every $i \in \{1, 2, \ldots, n\}$,

$$\mu_i(\sigma) \leq \mu_i(\sigma').$$

Lemma 1 shows a nice monotonicity property of SRPT. The next lemma is closely related.

Lemma 2 *(Lemma 4.3 in [16]) Let instance I' be obtained from I by removing some of the jobs from I. Let σ and σ' be SRPT schedules for, respectively, I and I' and let n, n' be the number of jobs in I and I'. Then, for every $i \leq n'$,*

$$\mu_i(\sigma) \leq \mu_i(\sigma').$$

Proof: For each job j that is included in I but not in I' we add a job j to I' with $r'_j = r_j$ and $p'_j = \infty$ (or some large enough number). In the SRPT schedule for the extended instance, the added jobs will complete last and the other jobs are scheduled as in σ'. Now the lemma follows directly from Lemma 1 with $\delta_j = 0$ for all j. (N.B. Phillips et al. [16] use the same argument. However, we do need the stronger version of Lemma 1 with arbitrary $\delta_j \geq 0$ to proof Lemma 4.) □

An advantage of unweighted completion times over weighted completion times is that we can use a volume argument. For example, in any feasible schedule, the sum of the last m completion times is bounded from below by the sum of all processing times. To approximate the sum of the last m completion times we may compare the total volume that SRPT has done until a moment t with the volume that could have been done by any other schedule. This backlog argument enables us to bound the sum of the last m completion times as we do in Lemma 4.

Given schedule σ, let $V_t(\sigma)$ be the volume processed until time t. Say that a schedule is *greedy* if at any moment, either all available jobs are being processed or all machines are busy. Any SRPT schedule is greedy. The next lemma gives an upper bound on the volume that a greedy schedule may do less than any other schedule. Figure 2 shows that the lemma is tight for $m = 2$.

Lemma 3 *Let*

$$\alpha = \sup_{I,t,\sigma,\sigma^*} \frac{V_t(\sigma^*) - V_t(\sigma)}{mt},$$

where σ is an arbitrary greedy schedule and σ^* is any feasible schedule, both for the same instance I on m machines. Then, $\alpha \leq 1/4$.

Proof: We give a short proof that $\alpha \leq 1/2$. Using this bound it follows that *SRPT* is at most 3/2-competitive. The stronger bound is included in the appendix. Consider an arbitrary time t and job j and assume the remaining processing time of j at time t is q_j in σ and $q_j^* \leq q_j - 1$ in σ^*. Then there are at least $q_j - q_j^*$ slots where j is processed in σ^* but not in σ. For each slot mark the position (time and place) in σ^*. Note that σ must process some other job at this position. Doing this for all jobs we see that the volume that σ processes before t is at least the total backlog. Hence, $V_t(\sigma^*) - V_t(\sigma) \leq V_t(\sigma)$, which implies

$$2(V_t(\sigma^*) - V_t(\sigma)) \leq (V_t(\sigma^*) - V_t(\sigma)) + V_t(\sigma) = V_t(\sigma^*) \leq mt.$$

Hence, $\alpha \leq 1/2$. □

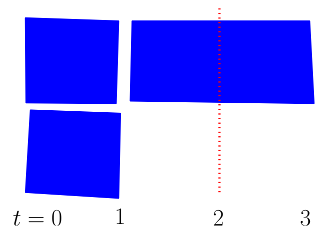

Figure 2: A tight example for Lemma 3. Take $m = 2$ and two jobs of length 1 and one job of length 2. All are released at time 0. It is possible to complete the jobs by time 2. The remaining volume at time $t = 2$ in the SRPT schedule is $1 = mt/4$.

Lemma 4 *Given an instance I with $n \geq m$ jobs, let τ be its SRPT schedule and ρ be an arbitrary feasible schedule for I. Then,*

$$\sum_{i=n-m+1}^{n} \mu_i(\tau) \leq \frac{5}{4} \sum_{i=n-m+1}^{n} \mu_i(\rho).$$

Proof: Let $t = \mu_{n-m}(\rho)$. We change the instance I into I' as follows such that no job is released after time t in the new instance. Every job j with $r_j \geq t+1$ gets release time $r'_j = t$ and processing time $p_j + r_j - t$. Let τ' be an SRPT schedule for I'. Then, by Lemma 1 we have

$$\mu_i(\tau) \leq \mu_i(\tau'), \text{ for any } i \in \{1, 2, \ldots, n\}. \tag{1}$$

On the other hand, we can change ρ into a feasible schedule ρ' for I' without changing any of the completion times since at most m jobs are processed after time t in ρ. Hence, we may assume

$$\mu_i(\rho) = \mu_i(\rho'), \text{ for any } i \in \{1, 2, \ldots, n\}. \tag{2}$$

Let $W_t(\tau')$ and $W_t(\rho')$ be the total remaining processing time at time t in, respectively, τ' and ρ'. Since the last m jobs complete at time t or later in ρ' we have

$$\sum_{i=n-m+1}^{n} \mu_i(\rho') \geq mt + W_t(\rho'). \tag{3}$$

Since no jobs are released after t, the SRPT schedule satisfies

$$\sum_{i=n-m+1}^{n} \mu_i(\tau') \leq mt + W_t(\tau'). \tag{4}$$

(Equality holds if τ' completes at least m jobs at time t or later than t.) By Lemma 3, $W_t(\tau') - W_t(\rho') = V_t(\rho') - V_t(\tau') \leq mt/4$. This combined with (3) and (4) gives

$$\begin{aligned} \sum_{i=n-m+1}^{n} \mu_i(\tau') &\leq mt + W_t(\tau') \\ &\leq \frac{5}{4} mt + W_t(\rho') \\ &\leq \frac{5}{4}\left(mt + W_t(\rho')\right) \leq \frac{5}{4} \sum_{i=n-m+1}^{n} \mu_i(\rho'). \end{aligned}$$

Equations (1) and (2) complete the proof. $\square$

Theorem 1 *SRPT is* 5/4*-competitive for minimizing total completion time on identical machines.*

Proof: Let φ be an optimal schedule. Take any $n' \leq n$ and let J' be the set of the first n' jobs completed in φ. Consider an SRPT schedule σ' for J'. By Lemma 2 we know that

$$\mu_i(\sigma) \leq \mu_i(\sigma') \text{ for all } i \leq |J'|. \tag{5}$$

We distinguish between the cases $n' \leq m$ and $n' \geq m$. In the first case we have $\mu_i(\sigma') \leq \mu_i(\varphi)$ since σ' starts each job at its release time and processes it without preemption. Combining this with (5) we get that

$$\mu_i(\sigma) \leq \mu_i(\varphi) \text{ for all } i \leq n'. \tag{6}$$

Now assume $n' \geq m$ and let φ' be the schedule φ restricted to jobs of J'. By definition,

$$\mu_i(\varphi') = \mu_i(\varphi) \text{ for all } i \leq |J'|. \tag{7}$$

We apply Lemma 4 with $\tau = \sigma'$ and $\rho = \varphi'$.

$$\sum_{i=n'-m+1}^{n'} \mu_i(\sigma') \leq \frac{5}{4} \sum_{i=n'-m+1}^{n'} \mu_i(\varphi'). \tag{8}$$

Using (5) and (7) we conclude that

$$\sum_{i=n'-m+1}^{n'} \mu_i(\sigma) \leq \frac{5}{4} \sum_{i=n'-m+1}^{n'} \mu_i(\varphi). \tag{9}$$

Hence, we see from (6) and (9) that the theorem follows by partitioning the completion times in groups of size m. The first group may be smaller. $\square$

2.1 More properties of SRPT

Given Lemmas 1 and 2 one might believe that a similar statement holds with respect to release times. However, it is not true that completion times do not decrease if release times are increased. In the example of Figure 1, SRPT will produce an optimal schedule if we change the release time of one small job from 0 to 1. The same example shows that SRPT may not be optimal even if no job is preempted. Finally, it is also not true that SRPT is optimal if it contains no idle time. This can be seen if we add two long jobs to example of Figure 1. This will not change the schedule of the other jobs and the sum of the completion times of the two long jobs is the same for SRPT and the optimal schedule. We conjecture that an SRPT schedule is optimal if it is non-preemptive and has no idle time.

3 Weighted jobs

The volume argument which is useful for bounding the average completion time becomes useless if jobs have arbitrary weights and we want to minimize the weighted average of completion times. A common approach is to use the *mean busy time* of a job which is defined as the average point in time that a job is processed. Given a schedule σ let $Z(\sigma)$ be the sum of weighted completion times and $Z^R(\sigma)$ be the sum of weighted mean busy times. On a single machine, the average (or total) weighted mean busy time is minimized by scheduling jobs preemptively in order of highest ratio of w_j/p_j [7]. This is called the preemptive *weighted shortest processing time* (WSPT) schedule. The WSPT-schedule is not unique but its total mean busy time is. Now consider a *fast single machine* that runs each job m times faster, i.e., job j has release time r_j and processing time p_j/m. For a given instance I, let $\sigma_m(I)$ be its preemptive WSPT-schedule on the fast single machine. The following inequality is a well-known lower bound on the optimal value of a preemptive and non-preemptive schedule [4, 20].

$$Z^R(\sigma_m(I)) + \frac{1}{2}\sum_j w_j p_j \leq \textsc{Opt}(I). \tag{10}$$

Our algorithm uses the same two steps as the algorithms by Schulz and Skutella [20] and Correa and Wagner [6]: First, the jobs are scheduled on the fast single machine and then, as soon as an α-fraction of a job is processed, a job is placed as early as possible on one of the parallel machines. The algorithm in [20] uses random values of α and a random assignment to machines. The deterministic algorithm of [6] optimizes over α and simply takes the first available machine for each job. Our algorithm differs at three points: First, we take a fast single machine schedule of a modified instance I' instead of I. Second, we do not apply preemptive WSPT but use non-preemptive WSPT instead. Third, we simply take $\alpha = 0$ for each job. The behavior of our algorithm depends on the input I and a real number $\epsilon > 0$.

Theorem 2 *With $\epsilon = 1/\sqrt{m}$, algorithm* $\textsc{Online}(\epsilon)$ *is δ_m-competitive for minimizing total weighted completion time, where $\delta_m = (1 + 1/\sqrt{m})^2(3e-2)/(2e-2)$. The ratio holds for preemptive and non-preemptive scheduling on m identical parallel machines.*

We denote the start and completion time of job j in the fast machine ρ_m by, respectively, s_j and c_j and in the parallel machine schedule ρ by S_j and C_j. First, we prove that the optimal value does not change much by the modification made in step (i).

Lemma 5 $\textsc{Opt}(I') \leq (1+\epsilon)\textsc{Opt}(I)$.

Proof: Let σ^* be an optimal schedule for I and for any job j let C_j^* be the completion time of j in σ^*. We stretch the schedule by a factor $1+\epsilon$ such that each job j completes at time $(1+\epsilon)C_j^*$ and starts at time

$$(1+\epsilon)C_j^* - p_j \geq (1+\epsilon)(r_j + p_j) - p_j = (1+\epsilon)r_j + \epsilon p_j \geq r_j'.$$

We see that the schedule is feasible for I' and its value is exactly $1+\epsilon$ times the optimal value of I. $\square$

Algorithm Online(ϵ):

INPUT: Instance $I = \{(p_j, w_j, r_j) \mid j = 1 \dots n\}$.

(i) Let $I' = \{(p'_j, w'_j, r'_j) \mid j = 1 \dots n\}$ with $p'_j = p_j, w'_j = w_j$ and $r'_j = r_j + \epsilon p_j$.

(ii) Apply *non-preemptive* WSPT to I' on the fast single machine. Let ρ_m be this schedule and let s_j be the start time of job j in ρ_m.

(iii) Each job j is placed at time s_j on one of the parallel machines as early as possible (but not before s_j). Let ρ be the final schedule.

Since we apply *non-preemptive* WSPT, the schedule ρ_m derived in step (ii) will in general not be the same as the fast single machine schedule $\sigma(I')$, which is derived by *preemptive* WSPT. Hence, we cannot use inequality (10) directly. We define a new instance I'' such that ρ_m is the fast machine schedule of I''. We shall proof this in Lemma 7 but first we introduce I'' and bound its optimal value like we did in the previous lemma. Let $I'' = \{(p''_j, w''_j, r''_j) \mid j = 1 \dots n\}$ with $p''_j = p_j, w''_j = w_j$ and $r''_j = \min\{\gamma_\epsilon r'_j, s_j\}$, where $\gamma_\epsilon = 1 + 1/(\epsilon m)$.

Lemma 6 $\textsc{Opt}(I'') \leq (1 + 1/(\epsilon m))\textsc{Opt}(I')$.

Proof: The proof is similar to that of Lemma 5. Let σ' be an optimal schedule for I' and C'_j the completion time of j in σ'. We stretch the schedule by a factor γ_ϵ such that each job j completes at time $\gamma_\epsilon C'_j$ and starts at time

$$\gamma_\epsilon C'_j - p_j \geq \gamma_\epsilon(r'_j + p_j) - p_j = \gamma_\epsilon r'_j + (\gamma_\epsilon - 1)p_j \geq \gamma_\epsilon r'_j \geq r''_j.$$

We see that the schedule is feasible for I'' and its value is exactly $1 + \epsilon$ times the optimal value of I'. □

Clearly, $\textsc{Opt}(I) \leq \textsc{Opt}(I'')$ since we only shift release times forward. Combining Lemmas 5 and 6 we see that $\textsc{Opt}(I'') \leq (1 + 1/(\epsilon m))(1 + \epsilon)\textsc{Opt}(I)$. Choosing $\epsilon = 1/\sqrt{m}$ we obtain the following corollary.

Corollary 1

$$\textsc{Opt}(I) \leq \textsc{Opt}(I'') \leq \left(1 + \frac{1}{\sqrt{m}}\right)^2 \textsc{Opt}(I). \tag{11}$$

If we want to prove a bound on the competitive ratio of our algorithm only for large values of m, then we may just as well compare our schedule with the optimal schedule of I'' instead of I since $\textsc{Opt}(I'')/\textsc{Opt}(I) \to 1$ for $m \to \infty$. The next lemma states that the total mean busy time of ρ_m equals the total mean busy time of the preemptive WSPT-schedule of I'' on the single machine.

Lemma 7 $Z^R(\rho_m) = Z^R(\sigma(I''))$.

Proof: We show that schedule ρ_m is a preemptive WSPT schedule for I''. First, ρ_m is a feasible schedule for the fast single machine relaxation of I'' since, by definition, $r''_j \leq s_j$. Next we use $s_j \geq r'_j \geq \epsilon p_j$.

$$\begin{aligned} c_j/s_j &= (s_j + p_j/m)/s_j \\ &= 1 + p_j/(ms_j) \\ &\leq 1 + p_j/(m\epsilon p_j) \\ &= 1 + 1/\sqrt{m}. \end{aligned} \tag{12}$$

Assume that at moment t, job j is being processed in ρ_m and job k is available in I'', i.e., $r''_k \leq t$. Denote $\gamma = 1 + 1/\sqrt{m}$, then by definition $r''_k = \min\{\gamma r'_k, s_k\}$. Since also $r''_k \leq t < s_k$ we must have $r''_k = \gamma r'_k$. Using (12) we get

$$r'_k = r''_k/\gamma \leq t/\gamma < c_j/\gamma \leq (1 + 1/\sqrt{m})s_j/\gamma = s_j.$$

We see that job k was available at the time we started job j in step (ii). Hence, we must have $w_k/p_k \leq w_j/p_j$. □

We apply the lower bound of (10) to instance I''.

$$Z^R(\sigma_m(I'')) + \frac{1}{2}\sum_j w_j p_j \leq \textsc{Opt}(I''). \tag{13}$$

Combining this with Corollary 1 and Lemma 7, we finally get a useful lower bound on the optimal solution.

Corollary 2

$$Z^R(\rho_m) + \frac{1}{2}\sum_j w_j p_j \leq \left(1 + \frac{1}{\sqrt{m}}\right)^2 \textsc{Opt}(I).$$

The lower bound of Corollary 2 together with the obvious lower bound $\textsc{Opt}(I) \geq \sum_j w_j p_j$ results in the following lemma.

Lemma 8 *Let $1 \leq \alpha \leq 2$. If $S_j \leq \alpha s_j$ for every job j, then*

$$\sum_j w_j C_j \leq \left(1 + \frac{\alpha}{2}\right)\left(1 + \frac{1}{\sqrt{m}}\right)^2 \textsc{Opt}(I).$$

Proof: Let b_j be the mean busy time of j in ρ_m, then $s_j = b_j - p_j/(2m) < b_j$.

$$\begin{aligned} C_j &= S_j + p_j \\ &\leq \alpha s_j + p_j \\ &< \alpha b_j + p_j \\ &= \alpha(b_j + p_j/2) + (1 - \alpha/2)p_j \end{aligned}$$

Next, we add weights and take the sum over all jobs.

$$\textstyle\sum_j w_j C_j \quad \leq \quad \alpha\left(Z^R(\rho_m) + \frac{1}{2}\sum_j w_j p_j\right) + (1 - \alpha/2)\sum_j w_j p_j$$

Now we use Corollary 2 and use that $\textsc{Opt}(I'') \geq \textsc{Opt}(I) \geq \sum_j w_j p_j$. For any $\alpha \leq 2$ we have

$$\begin{aligned}\sum_j w_j C_j &\leq \alpha(1+1/\sqrt{m})^2 \textsc{Opt}(I) + (1-\alpha/2)\textsc{Opt}(I) \\ &\leq (1+\alpha/2)(1+1/\sqrt{m})^2 \textsc{Opt}(I).\end{aligned}$$

□

First we give a short proof that $\alpha \leq 2$. This shows that the competitive ratio is at most $2 + o(m)$.

Lemma 9 *$S_j \leq 2s_j$ for any job j.*

Proof: Consider an arbitrary job j. At time s_j, the total processing time of jobs k with $s_k < s_j$ is at most ms_j. Since these are the only jobs processed on the parallel machines between time s_j and S_j we have $ms_j \geq m(S_j - s_j)$. Hence, $S_j \leq 2s_j$. □

The bound of the next lemma is stronger. The proof is given in the appendix. Lemma 8 tells us that the competitive ratio is at most $1 + \frac{e}{2(e-1)} \approx 1.791$ in the limit.

Lemma 10 *$S_j \leq \frac{e}{e-1} s_j$ and this bound is tight.*

3.1 Removing the $o(m)$

We can easily get rid of the $o(m)$ term at the cost of a higher ratio. Correa and Wagner [6] give a randomized α_m-competitive algorithm for the preemptive problem and a β_m-competitive algorithm for the non-preemptive version, where $2 - 1/m = \alpha_m < \beta_m < 2$ for $m \geq 3$. Let δ_m be our ratio as defined in Theorem 2. Then $2 - 1/m > \delta_m$ for $m \geq 320$. Hence, we get a randomized $2 - 1/320 < 1.997$-competitive for the preemptive version when we apply our algorithm for $m \geq 320$ and the α_m-competitive for $m < 320$. The ratio for the non-preemptive version is even closer to 2 (but strictly less than 2).

4 Conclusion

We have shown that approximation ratios less than 2 can be obtained for parallel machines by simple and efficient online algorithms. The lower bounds indicate that competitive ratios close to 1 are possible for randomized algorithms, especially when preemption is allowed.

Our analysis for SRPT is tight and it seems that a substantially different proof is needed to get below 1.25. Already, the gap with the lower bound, 1.105, is quite small. Muthukrishnan et al.[15] show that SRPT is at most 14 competitive w.r.t. the average stretch of jobs. Possibly, our result can reduce this ratio substantially.

The analysis for algorithm Online is not tight and a slight modification of the algorithm and analysis may give a ratio $e/(e-1) + o(m) \approx 1.58 + o(m)$. Moreover, the analysis is not parameterized by m. A refined analysis will reduced the $o(m)$ for small values of m.

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Appendix: Proof of Lemma 1, 3, and 10

Lemma 1 *Let I and I' satisfy $J = J'$ and for each $j \in J$ satisfy $r'_j = r_j - \delta_j \geq 0$ and $p'_j \geq p_j + \delta_j$, for some integers $\delta_j \geq 0$. Let σ and σ' be SRPT schedules for, respectively, I and I'. Then, for every $i \in \{1, 2, \ldots, n\}$,*

$$\mu_i(\sigma) \leq \mu_i(\sigma').$$

Proof We proof it by induction on the makespan of σ. Let $q_j(t)$ and $q'_j(t)$ be the remaining processing time of job j in, respectively, σ and σ' at time t. Define the multiset $Q(t) = \{q_j(t) \mid r_j \leq t\}$, i.e., it contains the remaining processing times of all jobs released at t or earlier. Let $Q'(t)$ contain the remaining processing times of the same set in σ', i.e., $Q'(t) = \{q'_j(t) \mid r_j \leq t\}$. Note that we take r_j and not r'_j in Q'. Let $Q_i(t)$ and $Q'_i(t)$ be the i-th smallest element in, respectively, $Q(t)$ and $Q'(t)$. We claim that for any time point t,

$$Q_i(t) \leq Q'_i(t), \text{ for all } i \in \{1, 2, \ldots, |Q_i(t)|\}. \tag{14}$$

If we can show (14) then the proof follows directly since $\mu_i(\sigma)$ ($\mu_i(\sigma')$) is the smallest t such that $Q_i(t)$ ($Q'_i(t)$) has at least i zero elements.

The proof is by induction on t. It is true for $t = 0$ since $Q(0) = Q'(0)$. Now consider an arbitrary time t_0 and assume the claim is true for and $t \leq t_0$.

First we analyze the changes when no job is released at time $t_0 + 1$. If σ processes less than m jobs in slot t then all non-zero elements in $Q(t_0)$ are reduced by one, implying, $Q_i(t_0 + 1) \leq Q'_i(t_0 + 1)$ for all $i \leq |Q_i(t_0 + 1)|$. Now assume σ processes less than m jobs in slot t_0. Then it processes jobs with remaining processing times $Q_{k+1}(t_0), Q_{k+2}(t_0), \ldots, Q_{k+m}(t_0)$ for some $k \geq 0$ while $Q_j(t_0) = 0$ for any $j \leq k$. Since $Q'_{k+1}(t_0), Q'_{k+2}(t_0), \ldots, Q'_{k+m}(t_0)$ are also non-zero, only values $Q'_s(t_0)$ with $s \leq k+m$ are reduced for σ'. Again, $Q_i(t_0 + 1) \leq Q'_i(t_0 + 1)$ for all $i \leq |Q_i(t_0 + 1)|$.

Now assume some jobs are released at time $t_0 + 1$. We may use the analysis above and only consider the affect of the newly added jobs. For any new job j we have $p_j = q_j(t_0 + 1) \leq q'_j(t_0 + 1)$. Clearly, (14) remains valid after the addition of these jobs. □

Lemma 3 *Let*

$$\alpha = \sup_{I,t,\sigma,\sigma^*} \frac{V_t(\sigma^*) - V_t(\sigma)}{mt},$$

where σ is an arbitrary greedy schedule and σ^* is any feasible schedule, both for the same instance I on m machines. Then, $\alpha \leq 1/4$.

Proof Given a schedule we say that a machine is *idle* in slot t if it is not processing any job in that slot. The *idle time* in slot t is the number of machines idle in that slot. We say that a slot is *idle* if at least one machine is idle in that slot.

We consider an arbitrary time T and show that $V_T(\sigma^*) - V_T(\sigma) \leq mT/4$. First we show that we may assume without loss of generality that σ^* has no idle time before time

T. Fill the idle time in σ^* before T with dummy jobs of unit length and let each such job be released at its start time. Now, take σ and add the same dummy jobs in the available idle time in a greedy way, i.e., as early as possible. The resulting schedule is still greedy. For any $t \leq T$, the increase for $V_t(\sigma)$ is no more than the increase for $V_t(\sigma^*)$ since the dummy jobs in σ are not placed earlier than in σ^*. Hence, the backlog can only increase using these dummy jobs. We assume from now that $V_T(\sigma^*) = mT$.

Our proof is by induction on T. If $T = 1$ then $V_T(\sigma) = m$ since σ is greedy and at least m jobs are available at time 0. Hence, $V_T(\sigma^*) - V_T(\sigma) = 0$. Now let $T \geq 2$ and assume $V_t(\sigma^*) - V_t(\sigma) \leq mt/4$ for any $t \leq T - 1$.

Let A be the set of jobs processed in slot T in σ. If $|A| = m$ then, by induction, $V_T(\sigma) = V_{T-1}(\sigma) + m \geq \frac{3}{4}m(T-1) + m > \frac{3}{4}mT$. From now we assume $|A| \leq m - 1$. For any $t \in [T]$, let $x_t = 1$ if slot t is idle in σ and $x_t = 0$ otherwise. Let y_t be number of idle machines in slot t. Note that $x_t = 0$ if $y_t = 0$. Consider a job $j \in A$ and let q_j be its remaining processing time at time T. In each idle slot t with $r_j + 1 \leq t \leq T$, job j must be processed since it is available and the schedule is greedy. This implies

$$q_j \leq p_j - \sum_{t=r_j+1}^{T} x_t. \tag{15}$$

The remaining processing time of job j in σ^* at time T, say q_j^*, is at least $p_j + r_j - T$. With (15) we get,

$$q_j - q_j^* \leq T - r_j - \sum_{t=r_j+1}^{T} x_t. \tag{16}$$

Next, we replace the term r_j in (16). Since $j \in A = \sigma(T)$ we have $r_j \leq T - 1$. By induction, $\sum_{t=1}^{r_j} y_t \leq mr_j/4$. Hence,

$$q_j - q_j^* \leq T - \frac{4}{m}\sum_{t=1}^{r_j} y_t - \sum_{t=r_j+1}^{T} x_t. \tag{17}$$

Let $Q = \sum_{j \in A}(q_j - q_j^*)$. Since there is at least one machine idle in slot T in σ, we know that any job j that has (partially) been processed by σ^* before time T is either completed by σ by time $T - 1$ or is processed in slot T. Therefore, Q is an upper bound on the idle time in σ before time T, i.e.,

$$mT - V_T(\sigma) \leq Q. \tag{18}$$

We will find an upper bound on Q. If $|A| \leq m/4$ then by equation (16), $Q \leq |A|T \leq mT/4$. From now on we may assume $m/4 \leq |A| \leq m - 1$. From (17) we get

$$Q \leq |A|T - \sum_{j \in A}\left(\frac{4}{m}\sum_{t=1}^{r_j} y_t + \sum_{t=r_j+1}^{T} x_t\right). \tag{19}$$

To simplify notation we denote the right hand side of (19) by R. Hence $Q \leq R$. Let $A_t = \{j \in A \mid t \geq r_j + 1\}$. By changing the order of summation, we can rewrite R as

$$R = |A|T - \sum_{t=1}^{T}\left(\frac{4}{m}(|A| - |A_t)|y_t + |A_t|x_t\right).$$

We continue rewriting and define for all $t \in [T]$

$$z(t) = \begin{cases} 0 & \text{if } y_t = 0 \\ \frac{4}{m}(|A| - |A_t|) + |A_t|/y_t & \text{if } y_t \geq 1. \end{cases} \tag{20}$$

Then

$$R = |A|T - \sum_{t=1}^{T} y_t z_t. \tag{21}$$

Next, we give a lower bound on z_t which is independent of t. We plug in a general inequality. Let $a < 1$, then

$$\begin{array}{rcll} (2a-1)^2 & \geq & 0 & \Rightarrow \\ 4a^2 - 4a + 1 & \geq & 0 & \Rightarrow \\ a & \geq & (4a-1)(1-a) & \Rightarrow \\ a/(1-a) & \geq & 4a - 1. & \end{array}$$

We substitute $a = |A_t|/m$. (Note that $a < 1$ since $|A_t| \leq |A| \leq m - 1$.)

$$\frac{|A_t|}{m - |A_t|} \geq 4\frac{|A_t|}{m} - 1. \tag{22}$$

If $y_t \geq 1$ then $y_t \leq m - |A_t|$ since each job in A_t is processed in slot t. We use this together with (22) and the definition of z_t. If $y_t \geq 1$ then

$$\begin{array}{rcl} z_t & \geq & \frac{4}{m}(|A| - |A_t|) + \frac{|A_t|}{m - |A_t} \\ & \geq & \frac{4}{m}(|A| - |A_t|) + 4\frac{|A_t|}{m} - 1 \\ & = & \frac{4|A|}{m} - 1. \end{array}$$

Note that this value is non-negative since $|A| \geq m/4$. We substitute this in (21).

$$Q \leq R \leq |A|T - \left(\frac{4|A|}{m} - 1\right) \sum_{t=1}^{T} y_t. \tag{23}$$

The total idle time until time T is $\sum_{t=1}^{T} y_t = mT - V_T(\sigma)$. Equations (18) and (23) imply

$$mT - V_T(\sigma) \leq Q \leq |A|T - \left(\frac{4|A|}{m} - 1\right)(mT - V_T(\sigma)).$$

From this linear inequality we get $mT - V_T(\sigma) \leq mT/4$. Hence $\alpha \leq 1/4$. □

Lemma 10 $S_j \leq \frac{e}{e-1} s_j$ *and this bound is tight.*

Proof Fix an arbitrary job k and assume for simplicity that $s_k = 1$ and $S_k = \alpha > 1$. We may do this if we assume processing times to be arbitrary rational numbers. For every

$i \in \{1, \ldots, k-1\}$ let V_i be the total processing time done on jobs $j \leq i$ between time s_{i+1} and α on the parallel machines. We give two bounds on V_{k-1} which results in an inequality for α.

Between time 1 and α all machines are busy since otherwise the algorithm would have started job k earlier. Further, only jobs $i \leq k-1$ are scheduled in this interval. This gives the following equation.

$$V_{k-1} = m(\alpha - 1). \tag{24}$$

Next, we give a different bound on V_{k-1}. We define $V_0 = 0$ and deduce a recursive bound on V_i for $i \geq 1$. Fix an arbitrary $i \in \{1, \ldots, k-1\}$. Job i becomes available for processing on the parallel machines at time s_i and the next job becomes available at time s_{i+1}. For any t let $\mu_i(t)$ be the number of machines working at time t on jobs from $\{1, 2, \ldots, i\}$. Since the algorithm places jobs in order of index and as early as possible, the value $\mu_i(t)$ is non-increasing in t for any $t > s_i$. Let $m' = \mu_i(s_{i+1})$. Then, by the monotonicity of μ,

$$V_i \leq (\alpha - s_{i+1})m'. \tag{25}$$

Since at least m' machines are busy from s_i till s_{i+1} and only jobs $j \leq i$ are processed we have

$$V_{i-1} + p_i \geq V_i + (s_{i+1} - s_i)m'. \tag{26}$$

Next, we combine (25) and (26) and use that $p_i = (c_i - s_i)m \leq (s_{i+1} - s_i)m$.

$$\begin{aligned}
V_{i-1} + (s_{i+1} - s_i)m &\geq V_i + (s_{i+1} - s_i)m' \\
&= V_i + \frac{s_{i+1} - s_i}{\alpha - s_{i+1}}(\alpha - s_{i+1})m' \\
&\geq V_i + \frac{s_{i+1} - s_i}{\alpha - s_{i+1}} V_i \\
&= \frac{\alpha - s_i}{\alpha - s_{i+1}} V_i.
\end{aligned}$$

Hence,

$$V_i \leq \mathcal{F}(V_{i-1}), \quad \text{with} \quad \mathcal{F}(V_{i-1}) = \left(\frac{\alpha - s_{i+1}}{\alpha - s_i}\right)(V_{i-1} + m(s_{i+1} - s_i)).$$

Note that $\mathcal{F}$ is monotone increasing in the argument (assuming all other values are fixed). Hence, if we define $W_0 = 0$ and $W_i = \mathcal{F}(W_{i-1})$ for $i \geq 1$ then $W_i \geq V_i$ for all $i \geq 1$ and in particular $W_{k-1} \geq V_{k-1}$. The recursion can easily be removed.

$$\begin{aligned}
W_{k-1} &= \sum_{i=1}^{k-1} m(s_{i+1} - s_i) \prod_{j=i}^{k-1} \frac{\alpha - s_{j+1}}{\alpha - s_j} \\
&= \sum_{i=1}^{k-1} m(s_{i+1} - s_i) \frac{\alpha - s_k}{\alpha - s_i} \\
&= m(\alpha - s_k) \sum_{i=1}^{k-1} \frac{s_{i+1} - s_i}{\alpha - s_i}.
\end{aligned}$$

Note that $s_k = 1$ and that for any $i \leq k-1$

$$\frac{s_{i+1} - s_i}{\alpha - s_i} \leq \int_{t=s_i}^{s_{i+1}} \frac{1}{\alpha - t}\, dt.$$

Hence,

$$W_{k-1} \leq m(\alpha-1) \int_{t=s_1}^{1} \frac{1}{\alpha - t}\, dt \leq m(\alpha-1) \int_{t=0}^{1} \frac{1}{\alpha - t}\, dt = m(\alpha-1) \ln \frac{\alpha}{\alpha-1}.$$

We combine this upper bound with equality (24).

$$m(\alpha-1) \leq m(\alpha-1) \ln \frac{\alpha}{\alpha-1} \quad \Rightarrow \quad e \leq \frac{\alpha}{\alpha-1} \quad \Rightarrow \quad \alpha \leq \frac{e}{e-1}.$$

Now we sketch a tight example. For simplicity we only give the single machines schedule with values s_i and p_i. Let $s_1 = 0, p_1 = e/(e-1)$ and for any $i \in \{2, \ldots, m\}$,

$$s_i = s_{i-1} + p_{i-1}/m \text{ and } p_i = e/(e-1) - s_i.$$

Note that is not really a possible realization of ρ_m (for example $s_1 > 0$ always holds), it is a tight example for the analysis and can be modified to make it valid realization.

Substituting p_{i-1} we get

$$s_i = (1 - \frac{1}{m}) s_{i-1} + \frac{e}{m(e-1)}$$

If $m \to \infty$ then

$$s_{m+1} \to \left(1 - \frac{1}{m} + \left(1 - \frac{1}{m}\right)^2 + \cdots + \left(1 - \frac{1}{m}\right)^m\right) \frac{e}{m(e-1)}$$

$$\to \left(\frac{\left(1 - \frac{1}{m}\right)^{m+1} - 1}{(1 - \frac{1}{m}) - 1}\right) \frac{e}{m(e-1)} \quad \to \quad \left(\frac{1/e - 1}{(-\frac{1}{m})}\right) \frac{e}{m(e-1)} = 1.$$

We see that the first m jobs are placed consecutively on the single machine in the interval $[0, 1]$. Each job starts on the parallel machine at its release time and ends at time $e/(e-1)$. At time 1, job $m+1$ becomes available and all machines are blocked until time $e/(e-1)$.$\square$

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